Incorporating price responsiveness into environmental lifecycle assessment

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Abstract

Lifecycle assessment(LCA) is a technique which is becoming increasingly important in environmental and energy policy discussions. Indicators like net energy and net carbon value of an energy source which figure in these discussions rely heavily on this technique. LCA today assumes linear technologies and produces outcomes that are numbers such as how many units of energy are needed to produce say a liter of ethanol fuel from a ton of corn. But as basic economics suggests, under reasonable conditions of some substitution between

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inputs and processes in production, this ratio is not a number but a function of prices. Changes in the price of economic goods is therefore likely to change the LCA outcome. Current LCA models are able to account for these changes only through explicit modification of the factor intensities used by the model. In this paper we illustrate the sensitivity of environmental indicators to economic conditions and then introduce a framework which can be used to predict LCA outcomes as a function of economic conditions. The main benefit of this approach is that LCA is more suitable for simulation of impact a carbon tax or other policies which in one way or another ultimately alter the relative prices of commodities.
Keywords: LCA, microeconomics, pollution, joint production, carbon tax, heterogeneity, biofuels

1 Introduction

A key argument in the societal debate against polices to support biofuels is that production of these alternative fuels may in fact consume more energy than they generate and emit more greenhouse gases than they sequester ([4], [9],[10],[11]). The analyses in support of those claims and counter-claims is based on a technique called lifecycle assessment (LCA). Metrics like net energy value and net carbon value that figure in these debates are derived using this technique. A central aspect of LCA is it assumes linear technologies and aims to obtain outcomes that are numbers - how many units of energy are needed to produce say a liter of ethanol fuel from a ton of corn. But as

basic economics suggests, under reasonable conditions of some substitutions between inputs and processes in production, this ratio not a number but a function of prices. Changes in the prices of economic goods is therefore likely to change the LCA outcome. Energy being an ubiquitous input to production a change in the relative price of different energy sources will induce adjustments in the form of fuel switching, substitution between capital, energy and labor etc. at several levels in the production chain of a commodity. This will obviously alter the lifecycle numbers. Current LCA outcomes change only if the physical parameters of the model such as quantity of coal or electricity used in production are altered. In other words current LCA is capable of answering, *how does a 10% decrease in the share of natural gas in the average electricity mix decrease the net carbon value of ethanol*? But it is not capable of answering, *if natural gas prices increase by 10% what is the impact on the net carbon value of ethanol*? Obviously the latter is more intuitive and useful way of framing the question than the former from a policy standpoint. The reality is physical that level of a factor adjusts in response to a change in the price of the factor which can happen because of a policy or a change in the market conditions. In this paper we introduce a framework which can be used to derive LCA indicators directly as a function of underlying economic parameters and thereby enable the use of LCA to simulate of the impact of policy actions on the environmental performance.

Section 2 provides some background on current LCA literature. Section 3 introduces a micro-economics based LCA that integrates prices directly into the lifecycle framework. We point out some implications of our model with

simple illustrations. Section 4 concludes the paper and describes directions for future work.

2 LCA Models

LCA is a systems approach to evaluating the environmental footprint of products, materials, and processes ([6],[7],[8]). The goal behind the development of LCA was to quantify the resource and environmental footprint of a product over its entire lifecycle from raw material extraction, manufacturing, and use till ultimate disposal. By resource footprint we mean the total physical flow of both extractive resources such as materials, energy, water etc. and polluting resources like green house gases, criteria air pollutants, toxic chemicals etc. through the various stages of the lifecycle. These physical values are then related to ultimate environmental burdens like global warming, acidification, smog, ozone layer depletion, eutrophication, deforestation etc. using established scientific relationships between emissions and impact. Depending on whether the physical accounting is done at an industry-wide level or whether it is done for a specific production chain there are two different types of LCA that exist today, namely, EIOLCA and process based LCA.

2.1 EIOLCA

EIOLCA computes the resource requirements and environmental emissions associated with production of a given value worth of a good, say, $1 million worth of steel or electricity. It does so by tracing out the various economic

transactions related to production like manufacturing, transportation, mining and related requirements etc. that would take place in order to produce the given value of the good. This information is derived from the economic input/output table of the economy a concept originally developed by Wassily Leontief. This model has been used extensively to calculate the environmental impact of major industrial products like steel, concrete, automotive fuels etc ([6]). But since the IO model represents fixed proportion (Leontief) production, it does not allow for substitution between inputs within a given sector. While this is not unreasonable in the short-run it is not a good assumption in the medium to long run. Sometimes even in the short-run farm operations offer scope for substitution between fossil energy and labor or between capital and energy. For example farmers may use less tilling or irrigation and use more land in response to higher energy prices. Such effects cannot be captured in this framework. Fertilizer and energy industries may also switch from gas to coal within the medium term in response to high oil prices or vice versa in response to a carbon pollution tax. Second, since the EIO table is aggregated across the whole economy, it captures the average effect rather than the marginal effects which are more important from a policy stand point.

2.2 Process LCA

The process approach to LCA is based primarily on the standard recommendations of the Society of Environmental Toxicology and Chemistry (SETAC) and emphasizes detailed modeling of each and every process in the produc-

tion chain (Hendrickson06). For example in the case of biofuel production, process LCA would distinguish between farming with irrigation and without irrigation, between farming with no-till, low-till and regular till, between inorganic and organic farming or between dry-mill and wet-mill fermentation of corn to ethanol etc. This approach is useful when analyzing the environmental impact of emerging products and technologies the effects of which are likely to be marginalized when one deals with industry wide aggregate data. For example, the LCA of cellulosic ethanol or the LCA of gasoline produced from tar sands is difficult to model using the EIO tables. Process LCA has been the main technique behind the major assessments of biofuels thus far ([4], [9], [11]).

3 Micro-economic LCA technique

The choice of inputs and technologies reflects producer behavior and therefore a good prediction should combine the technical and behavioral aspects of production. Economics develops a theory of production that allows choices and uses economic conditions to determine what exactly is selected. But current LCA either does not permit choices [[2]] or does not model behavior. In one of the earliest works that combine a *process LCA-like* material balance model with an economic model of production and consumption, Ayres and Kneese [1] outline a general equilibrium framework in which the flow of services, materials and pollutants are accounted for and related to welfare. The motivation for their model was the recognition that it is important to develop a method for forecasting pollution from a system wide perspective at

the scale of regional or national economy much like the motivation of LCA today. But the drawback of their approach too was that they assume fixed proportion for production within each sector. This limits the usefulness of having prices embedded in the model, since it does not permit any adjustment in input ratios as a function of relative prices. In reality there will be substitution and the example below shows the result of substitution between coal and gas.

Illustration of sensitivity of LCA outcome to assumptions about fuel mix Based on a meta-analysis of the various process LCA models of corn ethanol, Farrell et al.[4] report that on average each liter of corn ethanol produced in the US displaces 0.18 kilograms of carbon di-oxide equivalent emissions. This result is based on the assumption that the average conversion facility derives 60% of input energy from coal and 40% from natural gas. We performed sensitivity analysis of their model to various assumptions about the relative mix of coal and gas based energy input to corn conversion and fertilizer production. The results are shown in table [1]. In the extreme case when both biorefineries and fertilizer production shifts entirely to coal there is a net increase in GHG emissions from using corn ethanol compared to gasoline. On the other hand if say in response to a carbon tax the average facility shift entirely to natural gas then there is a 133% increase in the estimated lifecycle GHG benefit. This illustrates the importance of allowing for substitution possibilities in estimating lifecycle performance of biofuels.

Table 1: Sensitivity of ethanol LCA to fuel mix

	Scenario	kg of CO_2 eq. offset per liter of ethanol	% change over baseline
1	Baseline (Farrell et al *Science* 2006)	0.18	-
2	net GHG displacement if average biorefinery uses only coal based energy	0.09	-50%
3	net GHG displacement if average fertilizer production facility uses only coal based energy	0.07	-61%
4	net GHG displacement if both the average biorefinery and fertilizer producer use only coal	-0.01	-106%
5	net GHG displacement if average biorefinery uses only gas based energy	0.42	133%

3.1 A simple model of biofuel production

In this section we illustrate using a simplified representation of biofuel production (shown in figure [1]), how a parametric relationship between input prices and lifecycle emissions can be derived. We assume that ethanol, is produced with two inputs namely, corn and energy. Corn in turn is produced using two inputs namely, land and energy. Finally, energy can be produced from two different sources with different carbon intensities, say gas and coal. The former is generally considered a less polluting fuel relative to coal. For a variety of reasons, the elasticity of substitution between these two fuel sources may differ in each industry. In order to keep the mathematical exposition simple and intuitive we do not for now consider other essential inputs like capital or labor or other forms of energy. In the next section we generalize the results to an arbitrarily large model with more than two inputs

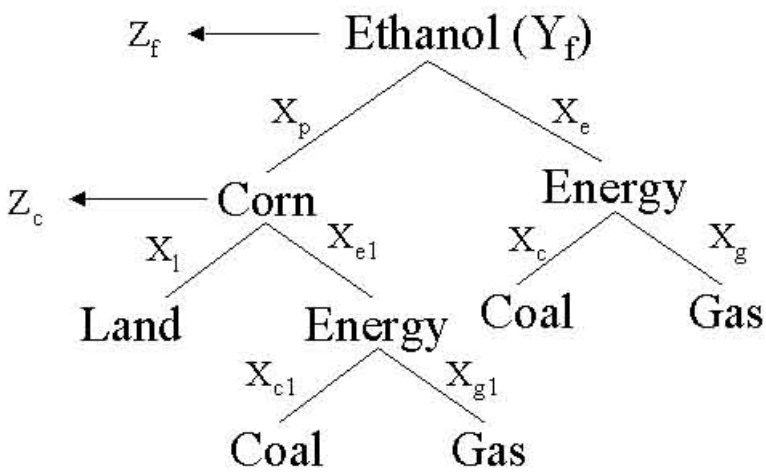

Figure 1: A simple model

and multiple stages in the lifecycle.

In figure [1],

Y_f - quantity of biofuel produced

X_p - quantity of plant matter required to produce the given quantity of biofuel

X_e - quantity of energy required to convert plant matter to biofuel

X_l - quantity of land to produce the required quantity of plant matter

X_{e1} - quantity of energy required to produce the required quantity of plant matter

X_c, X_g - quantity of coal and gas required to produce the required quantity energy for conversion

X_{c1}, X_{g1} - quantity of coal and gas required at the farm phase to produce plant matter

Z_p - pollution from farming

Z_f - pollution from conversion of plant matter of biofuel

Production functions The production relationships for the system shown in figure [1] are given by,

$Y_f = F_f(X_p, X_e)$

$X_p = F_p(X_1, X_{e1})$

$X_e = F_e(X_c, X_g)$ and $X_{e1} = F_e(X_{c1}, X_{g1})$

$Z_f = G_f(X_c, X_g)$

$Z_p = G_p(X_{c1}, X_{g1})$

Then $\Gamma_f = Z_f + Z_p$ denotes the total lifecycle emission associated with a quantity Y_f of biofuel while, $\frac{\Gamma}{Y_f}$ denotes the emission intensity of biofuel. But in reality corn is produced in much larger quantities than that used for biofuel production. Let us say a fraction a_{pf} of the total corn production is utilized for biofuel production. Then Z_p is a certain fraction of the total emissions associated total corn production in the economy i.e, $Z_p = a_{pf}\Gamma_p$, where Γ_p is the total lifecycle pollution associated with corn farming in the economy. Therefore,

$$\Gamma_f = Z_f + a_{pf}\Gamma_p \tag{1}$$

Assumptions about conditions of production We make the following assumptions about production conditions, namely,

1. profit maximizing behavior by producers
2. price taking behavior in both input and output markets by producers and
3. constant returns to scale (CRS) in production technology

Pollution generation function For simplicity we will assume that pollution arises solely from the use of energy and not other inputs. In other words, we ignore for now pollution that may result from land preparation processes like tilling or from conversion of land away from other previous uses. Also let the pollution function G be linear in inputs. That is,

$$G_f(X_c, X_g) = b_c * X_c + b_g * X_g \tag{2}$$

$$G_p(X_{c1}, X_{g1}) = b_c * X_{c1} + b_g * X_{g1} \tag{3}$$

where,

b_c, b_g are emission coefficients for coal and gas respectively.

Since $Z_f = G_f(X_c, X_g) = b_c * X_c + b_g * X_g$ differentiating with respect to p_c we get,

$$\frac{dZ_f}{dp_c} = b_c \frac{dX_c}{dp_c} + b_g \frac{dX_g}{dp_g} \tag{4}$$

Relationship between input price and emissions if production function is Cobb-Douglas Given these assumption we derive the mathematical relationships between the relative price of energy inputs and emissions from production of biofuel. The implications of relaxing one or more of these assumptions is described later. If production can be represented by a constant elasticity of substitution (CES) function we can write,

$Y_f = [(\alpha_p X_p)^{\rho_f} + (\alpha_e X_e)^{\rho_f}]^{1/\rho_f}$

$X_p = [(\alpha_l X_l)^{\rho_l} + (\alpha_{e1} X_{e1})^{\rho_l}]^{1/\rho_l}$

$X_e = [(\alpha_c X_c)^{\rho_1} + (\alpha_g X_g)^{\rho_1}]^{1/\rho_1}$

$X_{e1} = [(\alpha_{c1} X_{c1})^{\rho_2} + (\alpha_{g1} X_{g1})^{\rho_2}]^{1/\rho_2}$

where,

$\rho = \dfrac{1}{1-\sigma}$, and σ is the elasticity of substitution between inputs.

The CES functional form nests the common functional forms such as cobb-douglas (CD), linear production and the Leontief production functions. When the $\sigma = 1$, CES reduces to CD. We know that for a CD production function, the cost minimizing factor demands are given by,

$$x_i^* = x_i^*(\vec{p}_i, P, Y) = \frac{\alpha_i P Y}{p_i} \tag{5}$$

where,

x_i^* - optimal level of input use

$\vec{p}_i$ - vector of price of inputs

Y - quantity of output

P - output price

α_i - exponent for the ith input in the CD production function with $\sum_{i=1}^{n} \alpha_i =$

1 since we have assumed CRS technology

Differentiating equation [5] with respect to p_j we get,

$$\frac{dx_j^*}{dp_i} = \alpha_i \frac{x_j^*}{p_i} \text{ if } i \neq j \text{ \& } \frac{dx_i^*}{dp_i} = -(1-\alpha_i)\frac{x_i^*}{p_i} \tag{6}$$

Substituting for $\frac{dx_i^*}{dp_i}$ from equation [6] into [4]we get,

$$\frac{dZ_f}{dp_c} = -b_c(1-\alpha_c)\frac{X_c^*}{p_c} + b_g\alpha_c\frac{X_g^*}{p_c} \tag{7}$$

And substituting the expression for X_c^* and X_g^* from equation [5]we get,

$$\frac{dZ_f}{dp_c} = \alpha_c[\alpha_g\frac{b_g}{p_g} - (1-\alpha_c)\frac{b_c}{p_c}]\frac{P_f Y_f}{p_c} \tag{8}$$

A similar relationship can be derived for $\frac{dZ_p}{dp_c}$, the change in pollution for corn production with a change in price of coal.

Differentiating equation [1] with respect to p_c we get,

$$\frac{d\Gamma_f}{dp_c} = \frac{dZ_f}{dp_c} + \frac{d}{dp_c}(a_{pf}\Gamma_p) \tag{9}$$

If we assume a_{pf} the proportion of total corn output flowing into ethanol does not change, substituting [8] into [10] we get,

$$\frac{d\Gamma_f}{dp_c} = \alpha_c\frac{1}{p_c}[\alpha_g\frac{b_g}{p_g} - (1-\alpha_c)\frac{b_c}{p_c}](P_f Y_f + a_{pf}P_p X_p) \tag{10}$$

Similarly,

$$\frac{d\Gamma_f}{dp_g} = \alpha_g \frac{1}{p_g}[\alpha_c \frac{b_c}{p_c} - (1-\alpha_g)\frac{b_g}{p_g}](P_f Y_f + a_{pf} P_p X_p) \tag{11}$$

Equations [10] and [11] show the relationship between the change in lifecycle emissions and with a change in price of one of the inputs for a given level of output Y_f of biofuel. Given our assumptions about competitive behavior and CRS technology, input factor intensities are constant at any level of output and since pollution function is linear in inputs, therefore emissions scale linearly with level of output. If on the other hand the various factors are perfect substitutes then we can expect discrete shifts when relative price exceeds a certain threshold. Similar relationships can be derived for other production relationships like generalized CES, Leontief etc. The mathematics is straightforward but unwieldy and hence we do not derive it here.

3.1.1 Illustration of increase in fertilizer price on carbon emissions from corn production

Let us assume that corn is produced using four inputs, namely, land, fertilizer, irrigation and labor and that pollution is caused only from fertilizer use and irrigation only. We assume a cobb-douglas form for agricultural production and a linear pollution function.

$X_{corn} = f(X_L, X_F, X_I, X_l) = AX_L^{\alpha_L} X_F^{\alpha_F} X_I^{\alpha_I} X_l^{\alpha_l}$

$Z_{corn} = b_F * X_F + b_I * X_I$

where, X_{corn} - output of corn, Z_{corn} - carbon emissions due to corn produc-

Table 2: Sensitivity of ethanol LCA to fertilizer price doubles

Exponent for fertilizer in farm production (input parameter)	0.1	0.2	0.35
% increase in farm emissions when fertilizer price doubles	9%	17%	26%
% change in net GHG benefits per liter of ethanol	-11%	-17%	-22%

tion, L-land, F-fertilizer, I-irrigation, l-labor, α_i - elasticity of output for the ith input, b_F and b_I - carbon intensity of fertilizer and irrigation respectively.

For simplicity let us assume fertilizers are produced from natural gas while irrigation is using coal-based electricity. (This is not an unrealistic unassumption because more than 90% of fertilizers in US are gas-based while electricity in mid-west is dominated by coal and the carbon intensity of these inputs is the carbon intensity of natural gas and coal respectively. Ofcourse the results derived below hold only for irrigated corn production.) Using an expression similar to that in equation [10] and rewriting in terms of elasticities we can derive the % change in emissions from corn production. We then use these results in the model of Farrell et al. (Farrell06) to predict the % change in net GHG benefits of corn ethanol as fertilizer prices increase. Table [2] shows our estimates for three different elasticities of corn output to fertilizer input. Obviously since an increase in fertilizer price leads to substitution of fertilizer with other inputs which has a higher carbon intensity there is a net increase in lifecycle emissions and a decrease in the carbon offset by each liter of ethanol.

Table 3: Sensitivity of ethanol LCA to carbon tax on coal and gas

Carbon tax ($/ton)	5	10	15
% increase in relative coal price	17%	35%	57%
% change in net GHG benefits per liter of ethanol	117%	228%	383%

3.1.2 Pollution taxes

We now show how pollution taxes can be easily incorporated into our model. Let us say there is a tax of $\$\phi$ per unit of pollution Z rising from the production at any stage. Then the cost minimization problem can be written as

$$C = min(\vec{p}^T\vec{X} - \phi Z_k) = min((\sum_{j=1}^{n} p_j X_j) - \phi Z_k) \tag{12}$$

If pollution function Z is linear in inputs, i.e. $Z = \sum_{j=1}^{n} b_j X_j$ we can rewrite the above equation as

$$C = min \sum_{j=1}^{n} (p_j - \phi b_j) X_j = min \sum_{j=1}^{n} \tilde{p_j} X_j \tag{13}$$

where, $\tilde{p_j} = p_j - \phi b_j$, is the effective price received by producers

We now simulate the effects of a carbon tax on net GHG benefits of corn ethanol. Let us again assume that both corn production and conversion can be represented by a cobb-douglas production function. Table [3] shows the sensitivity of benefits for three different levels of carbon tax.

The model can thus be extended in a straight forward way to included policies like pollution taxes, production subsidies, import tariffs etc. all of which exist especially in the case of ethanol or biodiesel. The effect of a change in the

level of policy can therefore be easily analyzed by changing the coefficients in the model. Next we show how the simple model can be expanded to allow for joint production and decreasing returns to scale.

3.1.3 Joint production

In reality most production processes result in more than output i.e., they involve joint production. For example, the production corn ethanol by dry-milling yields substantial amount of a co-product called distiller's dried grains while production of biodiesel from oil seeds yields oil cake which is valued for it is use as a natural fertilizer. Similarly crop cultivation typically yields multiple outputs like grain, fodder and biomass for burning especially in developing countries. In such cases emissions arising from joint production should be allocated among the multiple products instead of being attributed entirely to one product, say the biofuel or the main crop. Mathematically if,

$$y_1 = f_1(x_1^1, x_2^1) \text{ and } y_2 = f_2(x_1^2, x_2^2)$$
$$Z = g(x_1, x_2) = b_1x_1 + b_2x_2 = b_1x_1 + b_2x_2, \text{ with,}$$
$$x_k = x_k^1 + x_k^2$$

where,

y_1, y_2 - joint outputs

f_1, f_2 - production function for y_1, y_2

x_1, x_2 - total quantities of factors 1 and 2 used in production

x_j^k - amount of factor j allocated to output k

We can then write, $Z = Z_1 + Z_2$ where, $Z_i = b_1x_1^i + b_2x_2^i$

So allocation of pollution among the various outputs would be relatively straight forward if we can observe the allocation of inputs to the various outputs and we can estimate the production functions assuming non-jointness. However such allocation of inputs is often unobserved or difficult to observe even if one wishes to. In other words the individual x_i^j's are unobserved and only $\sum_j x_i^j$ can be ascertained. However when individual allocation of inputs is unobserved, pollution can be allocated across the joint outputs in one of the following three ways, namely,

1. Allocation based on mass of output

$$Z_i = \frac{y_i}{\sum y_i} Z \tag{14}$$

2. Allocation based on market value of output

$$Z_i = \frac{p_i y_i}{\sum p_i y_i} Z \tag{15}$$

3. Allocation based on displacement of a competing production activity

$$Z_i = \gamma y_i \tag{16}$$

   where, γ is the pollution intensity of the activity that production of y_i displaces

Opinion is divided as to which is the appropriate method of allocation of pollution when there is joint production. In any case all of these can be

incorporated with little difficulty in our model.

3.1.4 Decreasing returns to scale

Non-constant returns to scale is a more realistic assumption in cases when there is heterogeneity. For example, if we consider heterogeneity in land or soil quality then as production expands into marginal lands the average productivity declines. Thus doubling biofuel production may entail a more than two fold increase in the amount of land under cultivation and/or the use of other inputs. And therefore the average emission per unit of biofuel produced will increase. Mathematically, DRS can be represented as

$$f(\lambda\vec{x}) \leq \lambda f(\vec{x}),\ \lambda \geq 1 \tag{17}$$

In the case of cobb-douglas production function, if the sum of the exponents $\sum_{i=1}^{n} \alpha_i < 1$ then this represents a DRS technology. Using duality theory and shepard's lemma we can show the following relationship between derived factor demands,

$$x_i^*(\lambda y, \vec{p}) > \lambda x_i^*(y, \vec{p}) \tag{18}$$

If $Z = \sum_{i=1}^{n} b_i x_i^*$ we can write,
$Z = Z(y, \vec{p}) = \sum_{i=1}^{n} b_i x_i^*(y, \vec{p})$

Similarly, $Z(\lambda y, \vec{p}) = \sum_{i=1}^{n} b_i x_i^*(\lambda y, \vec{p}) > \lambda \sum_{i=1}^{n} b_i x_i^*(y, \vec{p}) = \lambda Z(y, \vec{p})$

$\Rightarrow Z(\lambda y, \vec{p}) > \lambda Z(y, \vec{p})$

$\Rightarrow \dfrac{Z(\lambda y, \vec{p})}{\lambda y} > \dfrac{Z(y, \vec{p})}{y}$

Therefore average emissions per unit output will tend to increase when there are decreasing returns to scale and this is another advantage of using an approach that allows for non-linear production and pollution generation relationships.

3.2 The general model

In this section we extend the simple model described in section 3.1 to a more general setting which can include an arbitrarily large number of inputs and production stages. Let us assume there are n commodities and n production processes. For now we assume there is no joint production i.e, each production process results in one output and pollution. We illustrate later how this can be extended to include joint production. The total lifecycle emissions associated with production of a commodity can be defined as,

Total lifecycle emissions = Direct emissions from production + Indirect emissions from production of inputs

Mathematically,

$$\Gamma_k = Z_k + a_{1k} \times \Gamma_1 + .. + a_{1k} \times \Gamma_1 \ = Z_k + \sum_{i=1}^{n} a_{ik} \times \Gamma_i \ \ \forall k \in 1..n \qquad (19)$$

where,

Γ_k - total lifecycle emissions attributable to production of good k

Z_k - direct emissions at the point of production of good k

$a_{ik} \times \Gamma_i$ - indirect emissions attributable to production of good k

a_{ik} - share of total industry output of ith good used in production of kth good

n - total number of goods or production processes in the model

Rearranging equation [19]

$$-a_{1k} \times \Gamma_1 + .. + (1 - a_{kk}) \times \Gamma_k + .. - a_{nk} \times \Gamma_n = Z_k \, \forall k \in 1..n$$
$$\Rightarrow (I - A)\Gamma = Z$$

$$\Rightarrow \Gamma = (I - A)^{-1} Z \tag{20}$$

where,

$\Gamma = [\Gamma_1..\Gamma_n]^T$

$Z = [Z_1..Z_n]^T$

$I =$ identity matrix of size $n \times n$

$A = [a_{ik}]$ and a_{ik} is defined as

$$a_{ik} = \frac{X_{ik}}{Q_i} = \frac{X_{ik}}{Y_i + \sum_{j=1}^{n} X_{ij}} \, \forall i, k \in 1..n \tag{21}$$

where,

X_{ik} - output of industry 'i' used in the production of good 'k'

Q_i - total output of industry producing good 'i'

Y_i - final demand for good 'i'

$Q_i - Y_i =$ intermediate demand for good 'i'

This is ofcourse assuming that conditions under which $(I - A)^{-}1$ exists are satisfied.

Extending our definiton of pollution generation function to n inputs we have,

$$Z_k = g_k(X_{1k}, X_{2k}, ..X_{nk}) \, \forall k \in 1..n \tag{22}$$

where,

$g_k -$ is the pollution generation function for k^{th} industry

$X_{jk} -$ quantity of jth good used in production of kth good

Differentiating equation[19] with respect to p_i - the cost of the ith input

$$\frac{\partial \Gamma_k}{\partial p_i} = \frac{\partial Z_k}{\partial p_i} + \sum_{j=1}^{n} \frac{\partial (a_{jk} \times \Gamma_j)}{\partial p_i} \tag{23}$$

Differentiating equation[22] with respect to p_i

$$\frac{\partial Z_k}{\partial p_i} = \sum_{j=1}^{n} \frac{\partial g_k}{\partial X_{jk}} \frac{\partial X_{jk}}{\partial p_i} \tag{24}$$

$$\frac{\partial (a_{jk} \times \Gamma_j)}{\partial p_i} = \frac{1}{X_j} (\Gamma_j \frac{\partial X_{jk}}{\partial p_i} + X_{jk} \frac{\partial \Gamma_j}{\partial p_i}) \tag{25}$$

Note that in deriving the relationship for the simplified model we assumed a_{jk} was constant. We do not make that assumption here and hence we dif-

ferentiate the left hand side using chain rule to derive the above equation. Substituting equation[24] and equation[25] into equation[23] we get

$$\frac{\partial \Gamma_k}{\partial p_i} = \sum_{j=1}^{n} \left[\frac{\partial g_k}{\partial X_{jk}} \frac{\partial X_{jk}}{\partial p_i} + \frac{1}{X_j} \left(\Gamma_j \frac{\partial X_{jk}}{\partial p_i} + X_{jk} \frac{\partial \Gamma_j}{\partial p_i} \right) \right] \tag{26}$$

Rearranging,

$$-a_{1k} \times \frac{\partial \Gamma_1}{\partial p_i} + .. + (1 - a_{kk}) \times \frac{\partial \Gamma_k}{\partial p_i} + .. - a_{nk} \times \frac{\partial \Gamma_n}{\partial p_i} = \sum_{j=1}^{n} \left(\frac{\partial g_k}{\partial X_{jk}} + \frac{\Gamma_j}{X_j} \right) \frac{\partial X_{jk}}{\partial p_i} \tag{27}$$

Writing out the above equation $\forall k \in 1..n$ and rearranging in matrix notation we get,

$$\Rightarrow \frac{\partial \Gamma}{\partial p_i} = (I - A)^{-1} B \frac{d\vec{X}_k}{dp_i} \tag{28}$$

where,

$B = [b_{jk}]$ with $b_{jk} = \sum_{j=1}^{n} \left(\frac{\partial g_k}{\partial X_{jk}} + \frac{\Gamma_j}{X_j} \right)$

$\frac{\partial \Gamma}{\partial p_i} = \left[\frac{\partial \Gamma_k}{\partial p_1} .. \frac{\partial \Gamma_k}{\partial p_n} \right]^T$

$\frac{\partial \vec{X}_k}{\partial p_i} = \left[\frac{\partial X_{1k}}{\partial p_i}, .. \frac{\partial X_{jk}}{\partial p_i}, .. \frac{\partial X_{nk}}{\partial p_i} \right]^T$

3.3 Deriving the emission-price relationship from duality theory

Our ability to estimate these relationships depends on our ability to observe the production function F for each process or industry which ultimately depends on our ability to obtain data on input use. While this unlikely, we may however be able to estimate the dual cost function econometrically with just data for industry output and input prices which are more easily observed. We now show how the emission price relationships can be derived using duality theory. Of course this requires that the dual cost function satisfy a minimum set of regularity conditions [3] in addition to assumptions about producer behavior and technology.

Given an n input production function $F : Q = F(X_1, ..X_n)$, $X_i \geq 0$ and given a vector of input price $(p_1..p_n), p_i \geq 0$ and that producer does not exert monopsony power in input markets, the producer's cost minimization

$$C = min\, \vec{p}^T \vec{X} = \sum_{j=1}^{n} p_j X_j^* \tag{29}$$

$$\text{such that } F(X_1, X_2, ..X_n k) = Q \tag{30}$$

$$\text{with } Q = Y + \sum_{i=1}^{n} X_i \tag{31}$$

Setting up the lagrangian and solving for the optimal inputs we get,

$$X_j^* = X_j^*(Q, p_1, p_2, ..p_n) \tag{32}$$

Let C^k denote the indirect cost function for the kth industry. We can then write

$$C^k = C^k(X_1^*, ..X_n^*, p_1, ..p_n) = C^k(Q_k, p_1, p_2, ..p_n) \tag{33}$$

If the cost function satisfies the regularity conditions and is differentiable with respect to input prices, then using Shephard's lemma we can write,

$$\frac{\partial C^k(Q_k, p_1, p_2, ..p_n)}{\partial p_j} = X_{jk}^* \tag{34}$$

Thus knowing the industry output Q_k, input prices $\vec{p_i}$ and the industry cost function C^k we can in theory estimate the input demands X_{jk}^* for all j and k for every industry or every process we would like to model in the LCA. Now differentiating the equation again with respect to price of i_{th} input p_i we get,

$$\frac{\partial X_{jk}^*}{\partial p_i} = \frac{\partial^2 C^k(Q_k, p_1, p_2, ..p_n)}{\partial p_j \partial p_i} \tag{35}$$

We can in this way get the change in demand for any input with a change in price of any other input. We can write the vector of these changes in input demand as follows.

$$\frac{d\vec{X_{jk}}}{dp_i} = \vec{C_{ji}^k}(Q_k, p_1, p_2, ..p_n) = \left[\frac{\partial^2 C^k}{\partial p_1 \partial p_i}, .., \frac{\partial^2 C^k}{\partial p_j \partial p_i}, ..\frac{\partial^2 C^k}{\partial p_n \partial p_i}\right]^T$$

Substituting into equation,

$$\Rightarrow \frac{\partial \Gamma}{\partial p_i} = (I - A)^{-1} B \vec{C_{ji}^k} \tag{36}$$

Emission intensities can then be computed as follows

$$\Rightarrow \frac{\partial \frac{\Gamma}{Q}}{\partial p_i} = (I - A)^{-1} B \vec{C_{ji}^k} \tag{37}$$

where, $\frac{\Gamma}{Q} - [\frac{\Gamma_1}{Q_1}, \frac{\Gamma_2}{Q_2}, ..\frac{\Gamma_n}{Q_n}]^T$

We have now shown how the emission price relationship can in theory be derived using either the primal or dual relationships. Next we show how the basic model can be extended include scenarios like joint production and pollution taxes.

3.4 Input data requirements to the model

The input information required to operationalize our model are the following

1. Specification of production(or cost) functions for every process included within the the boundary of the system
2. Specification of the pollution function corresponding to each production process
3. The total industry output for each product
4. Prices of all commodities included within the model

3.5 Summary of the main steps in the analysis

Given the above information we list below the main steps involved in calculating the emission-price relationships

1. Identify the system boundary and the various production processes to be included within the boundary

2. For each process within the boundary identify the inputs and outputs

3. Define the production or cost relationships for each process within the boundary

4. Assume a production behavior, say profit maximization and determine the optimal level of inputs needed to produce a given level of output. Do this iteratively by starting with the ultimate product of interest in the analysis and then proceed all the way down till the most basic process in the production chain

5. Compute the A matrix. That is, determine the share of total output in a given industry that the level of outputs computed above comprise. For example if in the above step we determined that 2 million bushels of corn were needed to produce about 4.5 billion gallons of ethanol that the US produced in 2006 and we knew that the total corn production in the US was 10 million bushels, then the share of total corn production used for ethanol production is 0.2 or 20%

6. Specify the pollution generation function for each process

7. Write down equation [19] for each process and assemble the Γ matrix (equation [19])

8. Invert the Γ matrix to get the emission intensity equation as a function of input price and level of output for each process

9. Differentiate the emission intensity equation to get the derivatives of emission with respect to any input price

4 Conclusion and directions for future work

LCA has become a mature and useful tool for environmental impact assessment. But the numbers it generates has some limitations from a policy standpoint. Current LCA is suitable for evaluating the impact of replacing a chain of production activities with another rather than evaluate the impact of one or more policy actions. This limitation can be overcome by developing an LCA model which allows for choices in technology and by modeling producer behavior more explicitly. With this in mind we have introduced a new LCA model which predicts the lifecycle outcome as a function of prices. In deriving these relationships, we have relied on standard assumptions like well behaved neoclassical production and pollution functions and that capital is malleable and all inputs are generic. We first described the intuition and some implications of our approach using a simplified model. The formulation was then extended to a general setting involving an arbitrary number of inputs and stages of production. We also showed how we can model joint production, heterogeneity, non-constant returns to scale, etc. and how we

can simulate the effect of a policy like carbon tax using this general framework.

The main improvement over current LCA is the ability to predict the impact of changes in economic parameters on lifecycle indicators which should make it more useful for policy analysis. Although the emphasis has been on application to biofuel production and greenhouse gas emissions the framework is more broadly applicable to analysis of any industrial or agricultural production and other pollutants. As part of future work we hope to extend the model to allow for non-competitive behavior, risk and uncertainty and technology adoption. We also recognize that one another limitation is that ours is a partial equilibrium approach and so another area of future research is to incorporate general equilibrium effects. Empirical testing of the model with industry data also needs to be undertaken.

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